MATH 234 EXAM 2

Student Name:	Key I	Student Number:
Instructor:		Section:
Question 1.	(2 points each) Answer	by true or false:
1. Let A be an of A , then:	$m \times n$ matrix, let U be	e the row echelon form of A , and let F be the reduced row echelon form
(a)	All three matrices ha	ave the same row space.
(b) <u>F</u>	All three matrices ha	ave the same column space.
(c)	All three matrices ha	ave the same null space.
(d)	All three matrices ha	ave the same rank.
(e) <u>T</u>	_ All three matrices ha	ave the same nullity.
2 Le	t V is a vector space w	$ith dim(V) = 4, if \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4 \in V, then span \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} = V.$
3. <u> </u>	t V is a vector space wi	ith $\dim(V) = n$, let S be a subspace of V, then $0 < \dim(S) < n$.
4. <u>F</u> If	V is an infinite-dimensi	ional vector space, then any subspace of V is infinite-dimenstional.
5 If .	A is an $n imes n$ singular r	matrix, then $rank(A) < n$.
6. <u>F</u> Ev	ery linearly independen	at set of vectors in P_n must contain n polynomials.
7 If 1	V is a vector space with	$\dim(V) = n$, then any $n + 1$ vectors in V are linearly dependent.
8. <u>F</u> If	$\{v_1,v_2,\cdots,v_n\}$ are lines	arly independent in a vector space V , then V is finite-dimensional.
9 If 2	\mathbf{x}_1 and \mathbf{x}_2 are linearly in	ndependent in R^3 , then $\exists \mathbf{x} \in R^3$ such that $\operatorname{span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}\} = R^3$.
10 Th	e vectors e^x , xe^x , x are	e linearly independent in $C[0,1]$.
11. $\frac{1}{C[a,b]}$ Let	t $f,g,h\in C^2[a,b]$, if W	$V[f,g,h](x)=0$ for all $x\in [a,b],$ then f,g,h are linearly dependent in

Question 2 (3 points each) Circle the most correct answer:

- 1. One of the following is not a basis for P_3 :
 - (a) $\{1, 2x, x^2 x\}$
 - (b) $\{x-1, x^2+1, x^2-1\}$

 - (c) $\{x, x^2 + 3, x^2 5\}$ (d) $\{x^2 + 1, x^2 1, 2\}$
- 2. If V is a vector space with $\dim(V) = n$, then
 - (a) Any n linearly independent vectors in V span V.
 - (b) Any spanning set for V must contain at most n vectors.
 - (c) Any set containing less than n vectors must be linearly independent.
 - (d) All of the above.
- 3. In P_2 , the transition matrix corresponding to the change of basis from [1,x] to [x-1,x+1] is

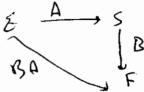
 - (b) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$
 - (c) $\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$
 - (d) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$
- 4. $\dim(\text{span}\{1-x, x^2, 3+x^2, 1+x^2\})$ equals
 - (a) 0
 - (b) 1
 - (c) 2
 - (d)3
- 5. The coordinate vector of $x^2 2x + 4$ with respect to the ordered basis $[1 x, 1 + x, x^2 2]$ is
 - (a) $(-4,2,1)^T$
 - (b) $(1,2,-4)^T$

6. If A is a 3×4 matrix, then

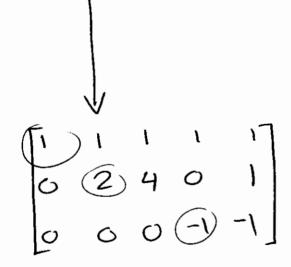
- (a) rank(A) = 3
- (b) rank(A) + nullity(A) = 3
- (c) nullity(A) = 3
- (d) none of the above.

7. Let V be a vector space. If A is the transition matrix from an ordered basis E of V to the standard basis and B is the transition matrix from the standard basis to an ordered basis F of V, then the transition matrix from F to E is





Question 3.(6 points) Let $x_1 = (2, 2, 2)^T$, $x_2 = (2, 4, 4)^T$, $x_3 = (2, 6, 6)^T$, $x_4 = (2, 2, 1)^T$, $x_5 = (2, 3, 2)^T$. Find a basis for \mathbb{R}^3 from the set $\{x_1, x_2, x_3, x_4, x_5\}$. Justify your answer.



Question 4.(12 points) Consider the vector space $R^{3\times3}$, which of the following is a subspace of $R^{3\times3}$? Justify your answer, i.e., if it is a subspace then prove it satisfies the conditions, if it is not then specify which condition is not satisfied with an example.

1.
$$S = \{A \in \mathbb{R}^{3 \times 3} \mid A^2 = A\}.$$

2.
$$S = \{A \in \mathbb{R}^{3 \times 3} \mid A \text{ is symmetric}\}.$$

3.
$$S = \{A \in \mathbb{R}^{3 \times 3} \mid A \text{ is triangular}\}.$$

Question 5.(15 points) Let
$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 3 & 11 & -1 \\ 1 & -1 & -5 & 1 \\ 2 & 0 & -4 & 2 \end{bmatrix}$$
.

1. Find a basis for the row space of A

2. Find a basis for the column space of A

3. Find a basis for the null space of A

|
$$|x| = t$$
, $|x| = s \implies |x| = (2t-s) - 3t$, $|t| = s$

| $|x| = t$, $|x| = s \implies |x| = (2t-s) - 3t$, $|t| = s$

| $|x| = t$, $|x| = s$

| $|x| = t$, $|x| = s$

| $|x| = t$, $|x| = s$

| $|x| = t$

| $|$

4. Find the rank and nullity of A

Question 6.(16 points) Let
$$E = [x, x + 2]$$
 and $F = [x + 3, x + 1]$ be two ordered bases for P_2 .

1. Find the transition matrix corresponding to the change of basis from E to the standard basis [1, x].

$$\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$$

2. Find the transition matrix corresponding to the change of basis from the standard basis [1, x] to F.

$$\begin{bmatrix} 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

3. Find the transition matrix corresponding to the change of basis from E to F.

4. Let p(x) = 3x - 11, find the coordinates of p with respect to F.

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{3} \end{bmatrix} \begin{bmatrix} -11 \\ 3 \end{bmatrix} = \begin{bmatrix} -7 \\ 10 \end{bmatrix}$$